1st European Mathematical Olympiad

Language: **English**

Day: **2**

Sunday, 26 April, 2026

Problem 5. Let $n \geqslant 4$ be a positive integer. Find all positive real numbers $x_1, x_2, \ldots, x_n$ such that

$$\begin{cases} x_1 + x_2 &= x_2x_3 + 1 \\ x_2 + x_3 &= x_3x_4 + 1 \\ &\vdots \\ x_{n-1} + x_n &= x_nx_1 + 1 \\ x_n + x_1 &= x_1x_2 + 1. \end{cases}$$

Problem 6. Determine all positive integers $n \geqslant 2$ with the following property: for every positive divisor d of n, the product of all the other positive divisors of n is a perfect power.

A perfect power is a number of the form a^b for some integers $a \geqslant 1$ and $b \geqslant 2$.

Problem 7. Let ABC be an acute triangle with $AB < AC$. Let M be the midpoint of segment BC. Let E and F be points on segments AC and AB, respectively, such that the circumcircle of triangle MEF is tangent to BC. The circumcircles of triangles AEF and ABC intersect at a point $P \neq A$. Let Q be a point on the circumcircle of triangle AEF such that AQ is perpendicular to BC.

Prove that PQ passes through the circumcentre of triangle MEF.

Problem 8. For a convex polygon $\mathcal{P}$, let $\mathcal{B}$ be the set of points on the boundary of $\mathcal{P}$. A function $f\colon \mathcal{B} \to \mathcal{B}$ is *European* if it satisfies the following properties:

(i) $f(f(X)) = X$ for all points $X \in \mathcal{B}$;

(ii) Line segments $Yf(Y)$ and $Zf(Z)$ have a common point strictly inside the polygon, for all points $Y, Z \in \mathcal{B}$.

What is the largest real number c such that for any convex polygon $\mathcal{P}$ and European function f, there is a point $W \in \mathcal{B}$ such that the length of line segment $Wf(W)$ is at least c times the perimeter of $\mathcal{P}$?

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Time: 4 hours and 30 minutes.
Each problem is worth 7 points.