1st European Mathematical Olympiad

Language: **English**

Day: **1**

Saturday, 25 April, 2026

Problem 1. Determine all positive integers n with the following property: the set $\{1, 2, \ldots, 2n-1, 2n\}$ can be partitioned into two disjoint sets $\mathcal{A}$ and $\mathcal{B}$ with n elements each, such that the sum of the elements of $\mathcal{A}$ divides the sum of the elements of $\mathcal{B}$.

Problem 2. Given a triangle ABC, let K and L be distinct points on side AC such that $\angle ABK = \angle CBL$. Rays BK and BL are not orthogonal to AC, and intersect the circumcircle of triangle ABC for the second time at points K_1 and L_1, respectively. Points K_2 and L_2 lie on the tangents to the circumcircle of triangle ABC at points K_1 and L_1, respectively, such that $\angle BKK_2 = \angle BLL_2 = 90°$.

Prove that points A, C, K_2, and L_2 lie on a circle.

Problem 3. Let $n \geqslant 2$ be an integer. Euroland has n cities, with direct flights connecting every pair of cities in both directions. For each pair of cities, the emperor assigns a positive price, which is the same in each direction. For two distinct cities A and B, let $D(A, B)$ be the number of flights in the cheapest journey between them; if there are multiple such journeys, then $D(A, B)$ is defined by the longest one.

For each value of n, find the largest possible average value of $D(A, B)$ over all pairs of distinct cities (A, B), that the emperor can achieve.

Problem 4. Let $\mathbb{N}$ be the set of positive integers. Find all functions $f\colon \mathbb{N} \to \mathbb{N}$ that simultaneously satisfy the following properties:

(i) $f(mn) = f(m)f(n)$ for all positive integers m and n;

(ii) There exists a positive integer c such that $f(n) \leqslant n^c$ for all positive integers n;

(iii) The numbers $f(n) + m$ and $f(m) + n + 1$ are coprime for all positive integers m and n.

Time: 4 hours and 30 minutes.
Each problem is worth 7 points.