Language: **English**

Day: **2**

Sunday, April 14, 2024

Problem 4. For a sequence $a_1 < a_2 < \cdots < a_n$ of integers, a pair (a_i, a_j) with $1 \le i < j \le n$ is called *interesting* if there exists a pair (a_k, a_ℓ) of integers with $1 \le k < \ell \le n$ such that

$$\frac{a_\ell - a_k}{a_j - a_i} = 2.$$

For each $n \ge 3$, find the largest possible number of interesting pairs in a sequence of length n.

Problem 5. Let $\mathbb{N}$ denote the set of positive integers. Find all functions $f\colon \mathbb{N} \to \mathbb{N}$ such that the following conditions are true for every pair of positive integers (x, y):

(i) x and $f(x)$ have the same number of positive divisors.

(ii) If x does not divide y and y does not divide x, then

$$\gcd(f(x), f(y)) > f(\gcd(x, y)).$$

Here $\gcd(m, n)$ *is the largest positive integer that divides both* m *and* n*.*

Problem 6. Find all positive integers d for which there exists a degree d polynomial P with real coefficients such that there are at most d different values among $P(0), P(1), P(2), \ldots, P(d^2 - d)$.